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Comparative study of pressure drop in multisized particulate slurry flow through pipe and rectangular duct

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Abstract

Reduction in energy requirement for hydraulic transportation of solids has been the main concern of researchers. Investigators have reported considerable reduction of head loss in rectangular ducts than traditional circular pipes. In the present study, concentration profiles for six particle sizes ranging from 38 to 739 μm were measured using a traversing mechanism and isokinetic sampling probe at nine levels in the vertical plane for multisized particulate zinc tailings slurry flowing through rectangular duct having hydraulic diameter of 80 mm, width of 200 mm and height of 50 mm. The experiments were conducted at different flow velocities ranging from 1 to 4 m/s using five efflux concentrations ranging from 4% to 26% by volume for each velocity. Solids concentration profiles were found to be a function of particle size, velocity of flow and efflux concentration of slurry. Solids concentration varied with the vertical position, except for particle size of 38 μm. Experimental data for pressure drop were also collected at six efflux concentrations ranging from 0% to 26% at flow velocities ranging from 1.0 to 4 m/s for each efflux concentration. Pressure drops and solids concentration profiles data measured in the present study for rectangular duct are compared with previous data for 105 mm diameter pipe.

Keywords: Solids concentration profiles; Pressure drop; Rectangular duct; Hydraulic transportation; Particle diffusivity; Particle size distribution; Friction factor

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1. Introduction

Circular pipes are normally used for long distance transportation of variety of materials in bulk quantities. Attempts have been made to make hydraulic transportation of solids as economical as practicable. One of the several aspects of economy is the power requirement. The two methods, which help to reduce the power required for slurry transport through pipeline are: the addition of polymer solution as suggested by Patterson et al. (1969) and fitting of helical ribs inside pipe (Charles, 1970). Polymer solutions are generally effective above a threshold Reynolds number. Provision of helical ribs may result in a considerable head loss reduction due to formation of helical or spiral flow, which is a combination of primary and secondary flow. In this system, the rate of solid transportation is very low and high initial costs combined with the problem of frequent maintenance put further limitation on this system.

Chiu and Seman (1971) have reported that square ducts may be used to replace the traditional circular pipes for the transport of solids. The cause for the reduction in head loss has been attributed to the formation of secondary currents. Prasad et al. (1980) compared experimentally the head loss for solid–liquid flow in a circular pipe with that of rectangular ducts of different aspect ratios a_r ($= w/h$, where w and h are width and height of the rectangular duct, respectively) keeping the cross-sectional area of each of them as same. They concluded that square duct shows a better performance in the low operating range of velocity, whereas a rectangular duct of a particular aspect ratio proved to be better from the energy requirement point of view for practically the entire velocity range over which the investigation was conducted. It was felt that not only the secondary flow inside a rectangular duct but also the larger bed area is a factor for head loss reduction.

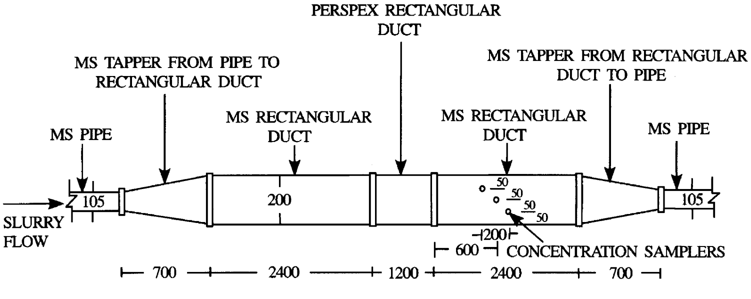
Ismail (1952) conducted experiments using water mixed with different proportions of sand through a rectangular duct having width 10.5 in. and 3 in. height. In terms of mean sedimentation diameter, two sizes of sand namely 0.10 and 0.16 mm were used. Jilan and Zhehuan (1988) studied experimentally the slurry flows through rectangular duct. Averbakh et al. (1997) measured velocity profiles in a rectangular duct to detect velocity fluctuations in the concentrated suspension induced with the help of Laser-Doppler anemometry. Shauly et al. (1997) studied experimentally the particle migration, velocity and concentration profiles in rectangular ducts for slow viscous flows of highly concentrated suspensions with the help of Laser-Doppler anemometry. Choi and Cho (2001) determined experimentally the effect of the aspect ratio of rectangular channels on the heat transfer and hydrodynamics of paraffin slurry flow. Ismail (1952) and Karabelas (1977) presented closed form expressions for predicting solids concentration profiles in the turbulent core of rectangular duct for dilute suspensions. Kaushal et al. (2002a) modified the Karabelas (1977) model for concentration and particle size distribution in the flow of multisized particulate slurry through rectangular duct by considering the effect of efflux concentration on particle diffusivity (ε_s) and settling velocity. On the basis of comparison with experimental data, Kaushal et al. (2002a) observed that modified Karabelas model shows good agreements at higher efflux concentrations also.

Among rectangular ducts, in case of rectangular ducts (aspect ratio $\geqslant 4.0$) having two dimensional slurry flow, it is easier to understand the flow and to develop the models for particle diffusivity and other parameters. In the present study, the experimental database required to develop correlation to predict the solids concentration profiles and pressure drop in a rectangular duct has been generated for the flow of multisized particulate slurry. The effect of efflux concentration and velocity on the composite and solids concentration profiles and pressure drop in rectangular duct and circular pipe has also been studied extensively.

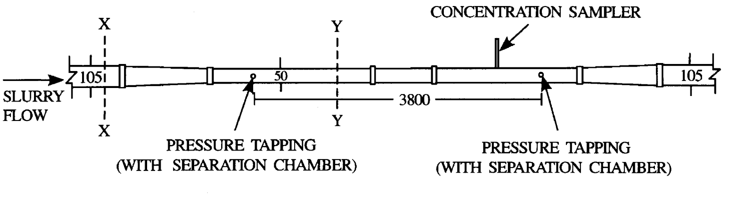
2. Experimental set up and instrumentation

The experiments were conducted using closed-loop pipeline fabricated from mild steel, with internal diameter of 105 mm and cross-sectional area of 0.008659 m^2. The details of the pipe loop are described elsewhere (Kaushal et al., 2002b). The pipe loop was modified by replacing a 7.4 m long pipe by a rectangular duct having hydraulic diameter of 80 mm, width of 200 mm, height of 50 mm and cross-sectional area of 0.01 m^2 as shown in Fig. 1. The length of duct was 6 m. The duct is made of 3 mm thick mild steel sheet. Two transition pieces, each of 0.7 m length were provided on the upstream and downstream of the duct to minimise flow disturbances.

For measurement of composite and solids concentration profiles and to check the two dimensionality of duct, three holes were provided on the wider side of the duct to enable measurement of concentration field by using a isokinetic sampling probe. These locations were selected at a distance of at least 4.1 m from the inlet of the duct so as to have a reasonably fully developed flow at the measurement locations. The spacing between the holes and from the wall was kept same (50 mm) as shown in Fig. 1. Composite concentration profiles were measured using



(a) Plan view



(b) Elevation

(c) Sections at XX and YY
All dimensions are in mm

Fig. 1. Schematic diagram of 200 mm × 50 mm cross-section rectangular duct used in the pilot plant test loop.

a sampling tube having a 4 mm × 6 mm rectangular slot, 3 mm above the end to collect representative samples at nine levels in the duct. Samples are collected from different heights from bottom of the duct in the vertical plane of the cross-section to determine the concentration profile under near isokinetic conditions. During the collection of concentration samples it was ensured that the flow of the slurry through the sampling tube outlet is continuous and uniform. If the tube got choked high pressure water was used to open it. Further sufficient time was allowed before sample collection in order to ensure steady state conditions. The sampling tubes were mounted on vernier type of traversing mechanism to enable traversing of the tube from the top to the bottom in vertical plane and its location can be accurately measured.

Solids concentration profiles of six different particle sizes in the duct at each velocity and efflux concentration are determined using the experimental procedure described by Kaushal and Tomita (2002).

Pressure tappings were installed in the side wall of rectangular duct at a distance of 3.8 m for pressure drop measurements. Separation chambers are provided at each pressure tap for interface separation of slurry and manometric fluid, water being the intermediate fluid. Pressure drop along the duct is measured by inverted manometer using air as the manometric fluid.

A 1.2 m long duct section was fabricated from 12 mm thick perspex sheet for facilitating the visualization of flow and the measurement of deposition velocity by observing the motion of the particles at the bottom of the duct. Magnetic flow meter was installed in the pipe loop for regular monitoring of the flow rate.

3. Material used and its properties

Zinc tailings obtained from a processing plant have been used to prepare the slurry for the present study. The average specific gravity was measured as 2.82. Particle size distribution in the fresh sample was determined after wet sieving 500 g of sample over 200 mesh BS sieve and is given in Fig. 2. Particles finer than BS 200 mesh were analysed using standard hydrometer technique.

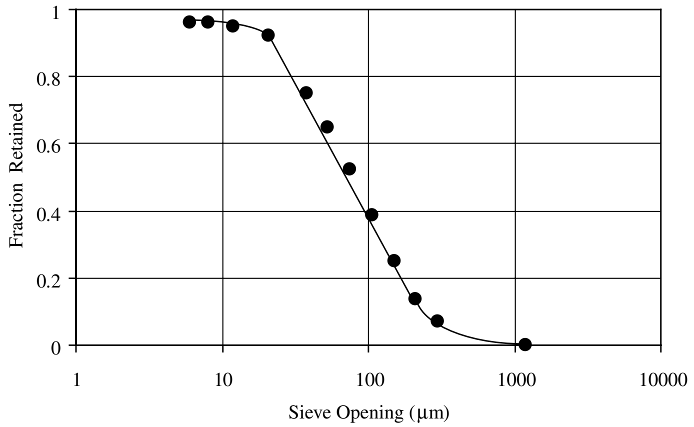


Fig. 2. Particle size distribution of zinc tailings used in the present study.

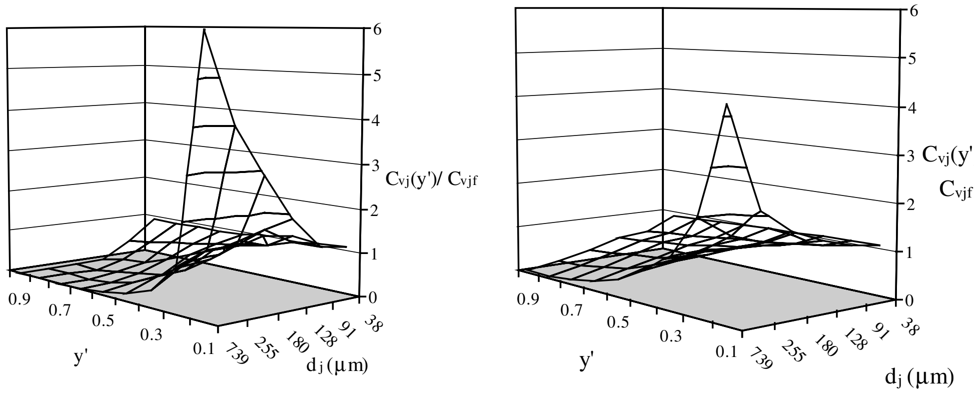
The settling behaviour of solid particles in suspension was determined by preparing a slurry having volumetric concentration of 13%. The slurry was allowed to settle in a 1000 ml jar and concentration of solids in the settled portion of slurry was observed at various intervals of time. It is seen that the zinc tailing slurry is slow settling and the final static settled concentration is 33.5% by volume.

4. Experimental results

Preliminary measurements for concentration were done at three vertical planes at the selected cross-section of the rectangular duct to check for the two dimensionality of the flow. It is seen that the concentration profiles at the three planes are nearly identical for different combinations of efflux concentration and velocity confirming the two dimensional nature of flow in the duct except near vertical walls. The experiments were conducted at different flow velocities ranging from 1 to 4 m/s using five efflux concentrations ranging from 4% to 26% by volume for each velocity. Fig. 3(a)–(e) show the extent of redistribution of particles of six different sizes at different flow velocities, where $C_{vj}(y')$ is the volumetric concentration of jth size particle at y' from bottom of the duct, $y' = y/h$, y is the distance from duct bottom, C_{vjf} is the efflux concentration by volume of jth size particle and d_j is the diameter of jth size particle. Generally we expect $C_{vj}(y')/C_{vjf}$ to increase from top to bottom of the duct. Further, for coarser size fractions, the ratio $C_{vj}(y')/C_{vjf}$ can be expected to vary considerably. It is observed that the values of $(C_{vj}(y')/C_{vjf})$ becomes unity at all heights for 38 μm size particle at all concentrations and velocities tested even at velocities close to the deposition velocity, thereby indicating this particle size distributed homogeneously across the duct cross-section. The other size fractions are asymmetrically distributed with the degree of asymmetry increasing with increase in particle size. It is also observed that the degree of asymmetry in the solids concentration profiles for same concentration of slurry increases with decreasing velocity. This is expected because with decrease in flow velocity there will be a decrease in turbulent energy, which is responsible for keeping the solids in suspension. From these figures, it is also observed that for a given velocity, increasing concentration reduces the asymmetry in the concentration profiles because of enhanced interference effect between particles. The effect of this interference is so strong that the asymmetry even at lower velocities is very much reduced at higher concentrations. Therefore it can be concluded that degree of asymmetry in the vertical concentration profiles depends upon the particle size, velocity of flow and concentration of slurry. Further, it is observed that the concentration profiles of coarser particles are highly skewed but as velocity increases the skewness in the concentration profiles tend to reduce and that all the particle sizes are distributed homogeneously (or $C_{vj}(y')/C_{vjf} = 1.0$) at the highest concentration irrespective of flow velocity. Similar observations for solids concentration profiles in the multisized zinc tailings slurry flow through 105 mm diameter pipe in a vertical plane have been made by Kaushal and Tomita (2002).

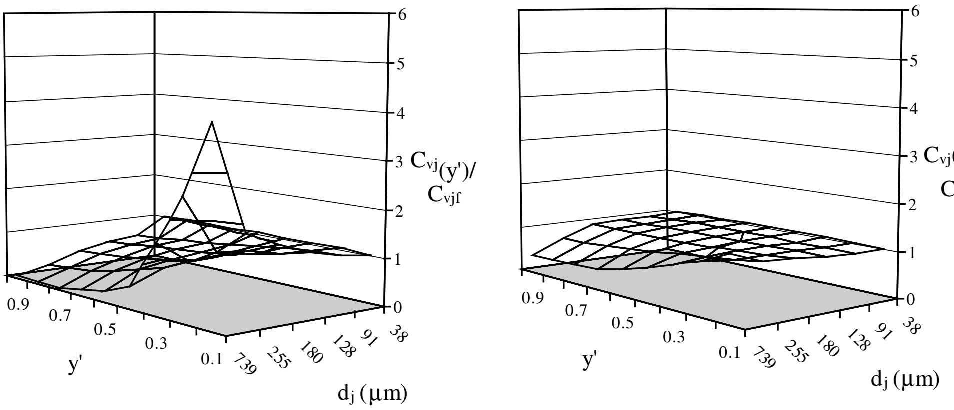
Weighted mean diameter profiles, $d_{wmd}(y')/d_{wmdf}$ vs. y' at different flow velocities using five efflux concentrations ranging from 4% to 26% by volume for each velocity have been calculated. The weighted mean diameter at a distance y' from the duct bottom, $d_{wmd}(y')$ is defined as

$$d_{wmd}(y') = \frac{\sum\{C_{vj}(y')d_j\}}{\sum C_{vj}(y')}; \quad j = 1, 2, \ldots, n \tag{1}$$



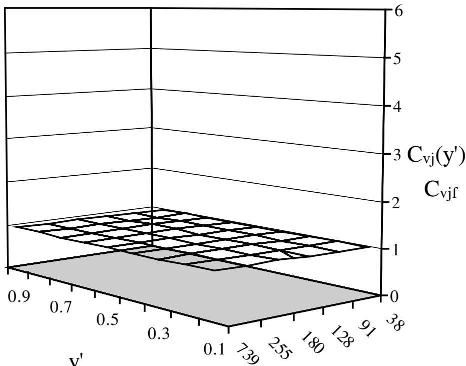
(a) $V_m = 1$ m/s, $C_{vf} = 3.9$ %

(b) $V_m = 2$ m/s, $C_{vf} = 8.3$ %



(c) $V_m = 2.75$ m/s, $C_{vf} = 12.6$ %

(d) $V_m = 3.5$ m/s, $C_{vf} = 19.6$ %



(e) $V_m = 4$ m/s, $C_{vf} = 26.0$ %

Fig. 3. Measured concentration profiles for different particle sizes in the flow of zinc tailings slurry through 200 mm × 50 mm cross-section rectangular duct with different efflux concentrations at different flow velocities.

The weighted mean diameter of the efflux sample, d_{wmdf} is defined as

$$d_{\mathrm{wmdf}} = \frac{\sum\{C_{\mathrm{v}j\mathrm{f}} d_j\}}{\sum C_{\mathrm{v}j\mathrm{f}}}; \quad j = 1, 2, \ldots, n \tag{2}$$

Composite concentration profiles, $C_{\mathrm{v}}(y')/C_{\mathrm{vf}}$ vs. y' at different flow velocities using five efflux concentrations ranging from 4% to 26% for each velocity have also been calculated, where $C_{\mathrm{v}}(y')$ is the composite volumetric concentration at y' from the duct bottom and C_{vf} is the efflux or overall average concentration.

It is observed that the degree of asymmetry in the composite concentration profiles and weighted mean diameter profiles increases with decreasing velocity for same efflux concentration. For a particular flow velocity, the degree of asymmetry decreases with increasing efflux concentration. Another important feature observed is that the concentration and weighted mean diameter of slurry at the centre line of the rectangular duct does not always correspond to the efflux values. Efflux concentration and efflux weighted mean diameter occur at some distance below the centre line of the rectangular duct. The general shape of the measured concentration profiles and weighted mean diameter profiles are similar to those obtained by Kaushal et al. (2002b) and Kaushal and Tomita (2002), respectively, in the multisized zinc tailings slurry flow through 105 mm diameter pipe.

In the literature, several studies (Graph et al. (1970); Shook (1976); Gillies and Shook (1991); Gillies et al. (2000); Schaan et al. (2000); Skudarnov et al. (2000) and Kaushal et al. (2002b)) regard deposition velocity as transition velocity between the sliding and stationary bed regimes for slurry flow through pipeline. The similar definition is followed in the present study for deposition velocity in the slurry flow through rectangular duct. In the present study, the deposition velocity test is carried out by initially allowing the slurry mixture to flow at high velocity such that homogeneous flow occurs. This condition that allows the solid particles to be uniformly distributed over the cross-section of the rectangular duct, is maintained until stable, steady state flow is achieved. The mixture velocity is then reduced in a gradual and controlled manner. As the mixture velocity is steadily reduced, the solids begin to drop out of suspension and form a thin moving bed of particles at the duct's bottom. The deposition velocity corresponds to the instant when these solid particles that have been forming the moving bed along the bottom of the duct stop moving, and begin to form a stationary bed.

Measurement of deposition velocity in the rectangular duct at five efflux concentrations ranging from 4% to 26% showed no significant change with efflux concentration. It is observed that deposition velocity increases only by a very little amount as efflux concentration increases. Deposition velocity varied from 0.90 to 0.98 m/s in the range of efflux concentration from 4% to 26%. Similar observations have been made by Schaan et al. (2000) in the flow of spherical glass beads, Ottawa sand and Lane mountain sand slurries. Schaan et al. (2000) found deposition velocity for spherical glass beads as constant over the range of solids concentrations from 5% to 45% by volume. For Ottawa sand and Lane mountain sand slurries, they reported only a little increase of around 0.2 m/s in deposition velocity in the range of solids concentrations from 5% to 30%. Kaushal and Tomita (2002) observed the deposition velocity to vary in the range from 1.1 to 1.18 m/s for zinc tailings slurry flow through 105 mm diameter pipe in the range of efflux concentration similar to the present study.

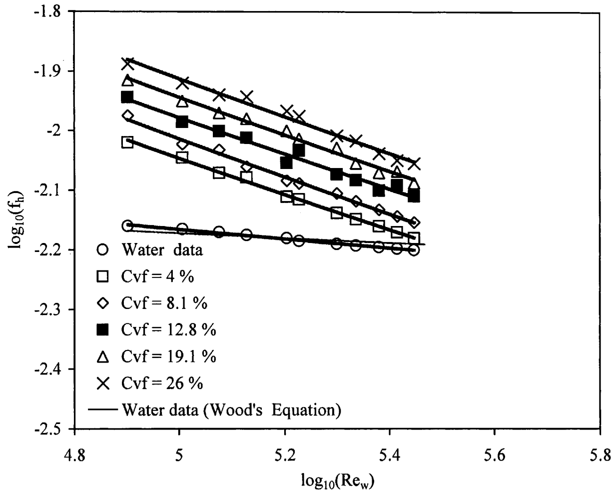


Fig. 4. Pressure drop data for the flow of water and slurry with different efflux concentrations through 200 mm × 50 mm cross-section rectangular duct.

Pressure drops over 3.8 m of duct length were measured for the multisized zinc tailings flowing through the duct at six efflux concentrations ranging from 0% to 26% using eleven flow velocities in the range from 1 m/s (slightly higher than the deposition discharge) to 3.5 m/s for each efflux concentration. The corresponding pressure drops for the multisized zinc tailings flowing through 105 mm diameter pipe have already been measured by Kaushal and Tomita (2002). The variation of Fanning friction factor f_h with Re_w for the multisized zinc tailings slurry flowing through rectangular duct and pipe at six efflux concentrations ranging from 0% to 26% is shown in Figs. 4 and 5, respectively. f_h is computed using the following formula:

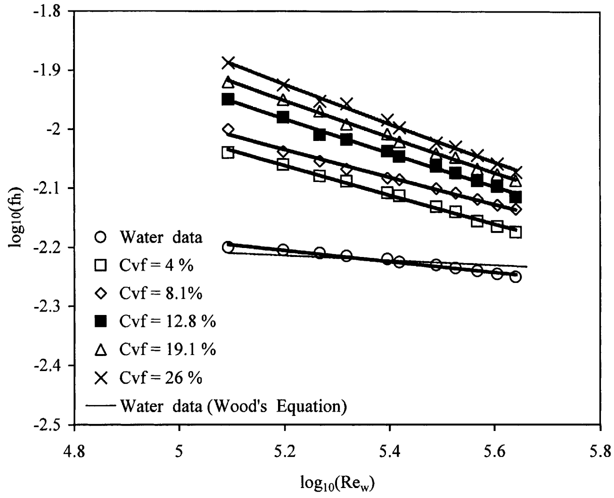


Fig. 5. Pressure drop data for the flow of water and slurry with different efflux concentrations through 105 mm diameter pipe.

$$f_{\mathrm{h}} = \frac{gD_{\mathrm{h}}i}{2V_{\mathrm{m}}^2} \tag{3}$$

where i is the pressure gradient in terms of m of water column per m length of the conduit, D_{h} is the hydraulic diameter defined as $D_{\mathrm{h}} = 4A/P_{\mathrm{w}}$ and is $4wh/(2w + 2h)$ for rectangular duct, where A is the cross-sectional flow area, P_{w} is the wetted perimeter, V_{m} is the flow velocity and g is the acceleration due to gravity. For the present study D_{h} for rectangular duct is 80 mm. Re_{w} is the Reynolds number calculated using following correlation:

$$Re_{\mathrm{w}} = \frac{\rho_{\mathrm{w}}V_{\mathrm{m}}D_{\mathrm{h}}}{\mu_{\mathrm{w}}} \tag{4}$$

where ρ_{w} and μ_{w} are the mass density and viscosity of water. It is observed that at low velocities the slurry pressure drops are slightly higher than the corresponding values for water and with increase in velocity, pressure drop increases more rapidly.

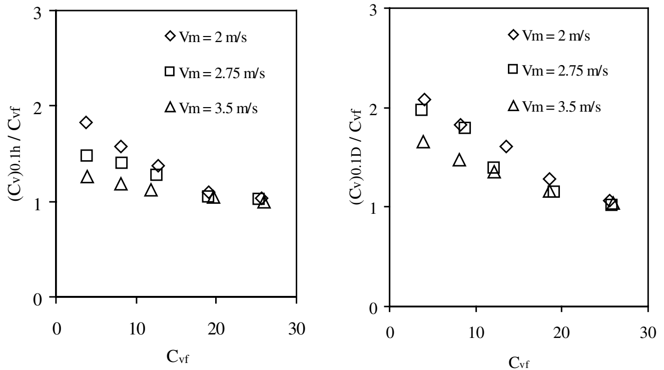
In order to check the accuracy of the experimental pressure drop in the flow of water, the results were compared with the Wood (1966) equation. Wood's equation expresses a relationship for friction factor as a function of relative roughness ($\varepsilon/D_{\mathrm{h}}$) and Reynolds number. Water data are found to follow the Wood's equation with $\varepsilon/D_{\mathrm{h}}$ as 0.0015 and 0.001 for rectangular duct and pipe, respectively, as shown in Figs. 4 and 5.

5. Comparison of solids concentration profiles and pressure drop in rectangular duct and pipe

5.1. Solids concentration profiles

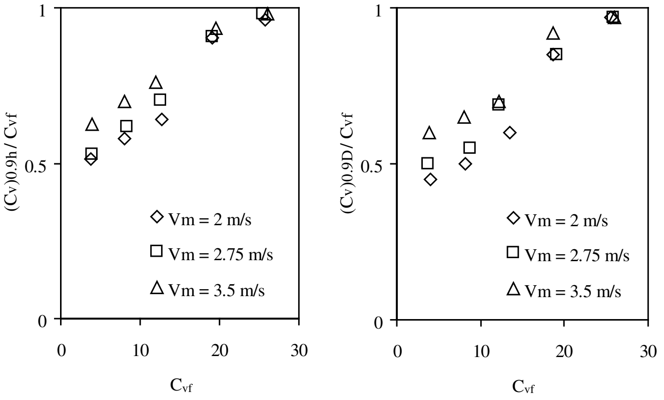
To quantify the effect of efflux concentration and velocity on the extent of asymmetry of concentration profile, the variation of normalised composite concentration in rectangular duct and pipe at the bottom [$(C_{\mathrm{v}})_{0.1h}/C_{\mathrm{vf}}$ and $(C_{\mathrm{v}})_{0.1D}/C_{\mathrm{vf}}$, where $(C_{\mathrm{v}})_{0.1h}$ and $(C_{\mathrm{v}})_{0.1D}$ are the volumetric composite concentration at $y' = 0.1$ in rectangular duct and pipe, respectively] and at the top [$(C_{\mathrm{v}})_{0.9h}/C_{\mathrm{vf}}$ and $(C_{\mathrm{v}})_{0.9D}/C_{\mathrm{vf}}$, where $(C_{\mathrm{v}})_{0.9h}$ and $(C_{\mathrm{v}})_{0.9D}$ are the volumetric composite concentration $y' = 0.9$ in rectangular duct and pipe, respectively] for zinc tailings with similar characteristics have been plotted against efflux concentration C_{vf} at three flow velocities of 2, 2.75 and 3.5 m/s using five efflux concentrations ranging from 4% to 26% for each velocity, as shown in Fig. 6. It is clearly seen that at any given velocity with increase in efflux concentration, the solids concentration at the bottom and top of the rectangular duct and pipe approaches unity which implies that the solids concentration reduces in the lower half and increases in the upper half. From these figures it is also seen that the rate of approaching homogeneity with increase in efflux concentration at lower velocity is faster than that at higher velocity because the normalised concentration reaches unity for all the three velocities almost at the same efflux concentration.

Kaushal and Tomita (2002) have reported the solids concentration profiles for six particle sizes, composite concentration profiles and weighted mean diameter profiles for zinc tailings through the pipe at three flow velocities of 2, 2.75 and 3.5 m/s using five efflux concentrations ranging from 4% to 26% for each velocity.



(a) at the bottom of rectangular duct

(b) at the bottom of pipe



(c) at the top of rectangular duct

(d) at the top of pipe

Fig. 6. Effect of efflux concentration and flow velocity on composite concentration profile at the bottom ($y' = 0.1$) and top ($y' = 0.9$) of the 200 mm × 50 mm cross-section rectangular duct and 105 mm diameter pipe.

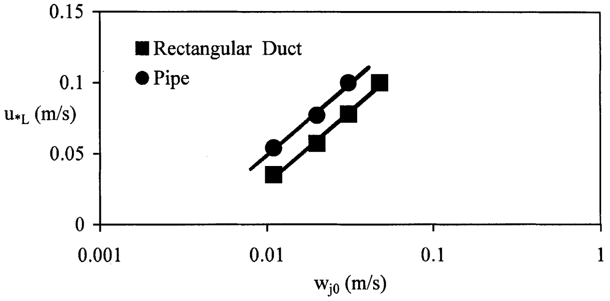
From the data collected in the present study and Kaushal and Tomita (2002) pipe data, it is observed that at the highest efflux concentrations of 26%, all the particle sizes are distributed homogeneously in rectangular duct and pipe at higher flow velocities. The coarsest particle has less asymmetric concentration profiles across the duct cross-section than that of pipe at corresponding flow velocities for efflux concentration of 26%. At other efflux concentrations, all the particle sizes except the finest particle (38 μm size) distribute asymmetrically across the cross-section with less asymmetricity in case of rectangular duct than that of pipe at the corresponding flow velocities, while the finest particle is homogeneously distributed across the cross-section in both the rectangular duct and pipe at all the flow velocities and efflux concentrations. Further, the composite concentrations are homogeneously distributed across the cross-section in rectangular duct and pipe at all the flow velocities at the highest efflux concentration 26%. The coarsest size particle, which is asymmetrically distributed at 26% for lower velocities has very little effect on composite concentration profile due to its low efflux concentration. At other efflux concentrations 4%, 8%, 13% and 19%, the composite concentration is distributed asymmetrically across the cross-section with less asymmetricity for rectangular duct than that for pipe at the corresponding flow velocities. It is observed that the weighted mean diameters are distributed asymmetrically at all the efflux concentrations with less asymmetricity across the cross-section of rectangular duct than that of pipe at the corresponding lower flow velocities. The coarsest size particle, which is asymmetrically distributed at 26% by volume has visible effect on weighted mean diameter profile due to its large size.

Furthermore, Fig. 6 shows that the asymmetricity of composite concentration profiles and weighted mean diameter profiles in rectangular duct at the flow velocities of 2 and 2.75 m/s is less than that in pipe even at flow velocities of 2.75 and 3.5 m/s, respectively, for most of the efflux concentrations.

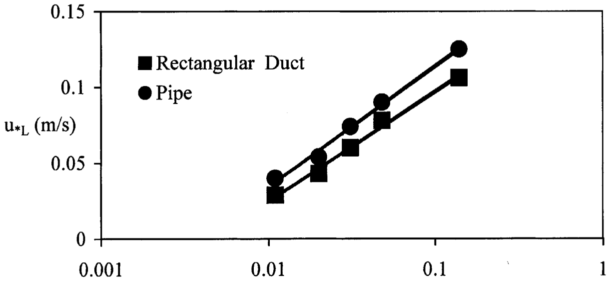
Detailed studies on turbulence phenomena in boundary layer flow by Hinze (1975) suggest that the maximum value of vertical turbulence velocity fluctuations is of the same order as the bed-shear velocity. Comparison between the required shear velocity, $u_{*\mathrm{L}}$ and the calculated terminal velocity (unhindered settling velocity, w_{jo}) of the particles to maintain the particles at constant volumetric concentration of 19% and 26% across the pipe and rectangular duct cross-section is shown in Fig. 7(a) and (b), respectively. $u_{*\mathrm{L}}$ is calculated using following correlation:

$$u_{*\mathrm{L}} = V_{\mathrm{L}}\sqrt{\frac{f_{\mathrm{hw}}}{8}} \tag{5}$$

where V_{L} is the velocity at the transition of homogeneous and heterogeneous regimes and f_{hw} is the friction factor corresponding to the measured pressure drop for equivalent discharge of clear



(a) $C_{vf} = 19$ %

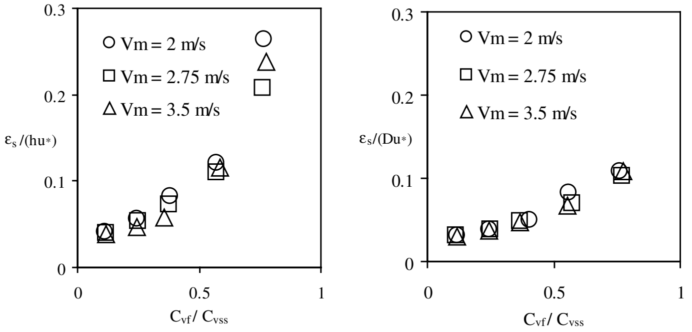


(b) $C_{vf} = 26$ %

Fig. 7. Comparison of terminal velocities of different particles and shear velocity required to maintain the constant concentration across the rectangular duct and pipe cross-section.

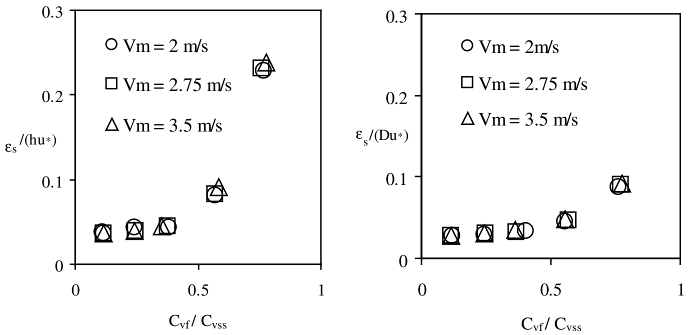
water. w_{jo} for different particles are calculated using standard drag relationships. It is observed that shear velocity required to maintain the particles at particular volumetric concentration is more in case of pipe. It increases with increase in size of particles due to gravitational effect. However, for a particular particle size, it decreases with increase in slurry concentration. Such a decrease in required shear velocity with increase in concentration may be attributed to the increased particle diffusivity, which helps particles to remain in suspension.

The particle diffusivity ε_s at the bottom ($y' = 0.1$) and top ($y' = 0.9$) at a particular efflux concentration and flow velocity is calculated using the methods described by Kaushal et al. (2002a) for rectangular duct and Kaushal and Tomita (2002) for pipe. The dimensionless particle diffusivity, ξ [$\varepsilon_s/(Du_*)$ for pipe and $\varepsilon_s/(hu_*)$ for rectangular duct] is shown in Fig. 8. From this figure, it is clear that particle diffusivity increases exponentially with efflux concentration. It is observed that particle diffusivity in rectangular duct is more than that in pipe for all the efflux concentrations. Thus the solid distribution across the pipe cross-section will be more asymmetric than that in rectangular duct according to the basic diffusion equation, which is borne out from experimental observations also. However, the difference in asymmetry of concentration distribution decreases with increase in efflux concentration, the rate of increase in the value of ξ increases with increase in efflux concentration as ξ becomes insensitive at higher efflux



(a) at the bottom of rectangular duct

(b) at the bottom of pipe



(c) at the top of rectangular duct

(d) at the top of pipe

Fig. 8. Effect of efflux concentration and flow velocity on particle diffusivity ε_s at the bottom ($y' = 0.1$) and top ($y' = 0.9$) of the 200 mm × 50 mm cross-section rectangular duct and 105 mm diameter pipe.

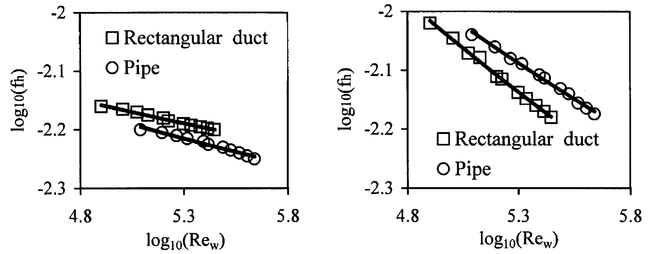
concentrations. Further, ξ was found to be insensitive to flow velocity at any given efflux concentration.

5.2. Pressure drop

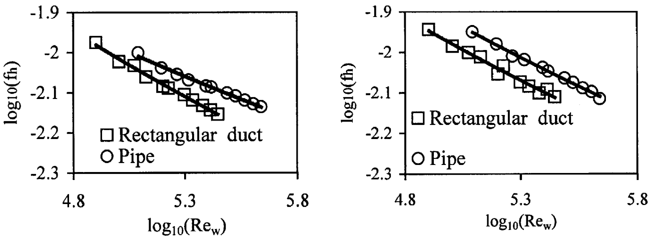
From Fig. 9(a), it is observed that friction factor in water flow through rectangular duct is always more than that for the pipe. The increase in pressure drop in case of rectangular duct can be attributed to its larger relative roughness than that of pipe. In contrary, from Fig. 9(b)–(f), the pressure drop in slurry flow through rectangular duct is always less than that for the pipe. The obvious reason for this decrease in pressure drop for slurry flow, in case of rectangular duct, must be the secondary flow, which helps in keeping the particles in suspension.

Pressure drop in slurry flow may be expressed as:

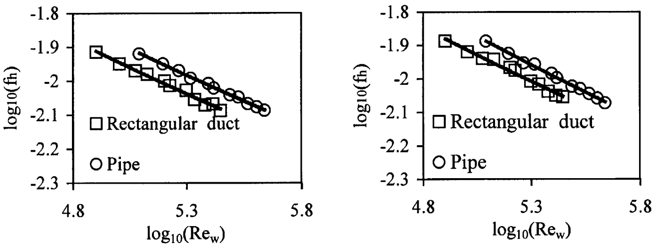
$$f_h = f_{hw} + f_{hs} \tag{6}$$



(a) Water data (b) $C_{vf} = 4$ %

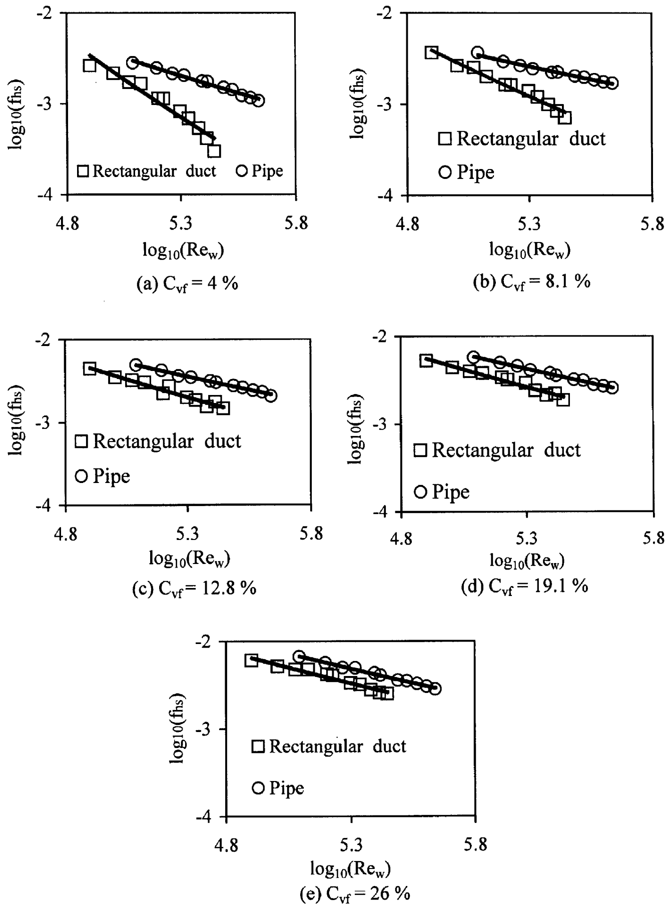


(c) $C_{vf} = 8.1$ % (d) $C_{vf} = 12.8$ %



(e) $C_{vf} = 19.1$ % (f) $C_{vf} = 26$ %

Fig. 9. Variation of friction factor f_h with Reynolds number Re_w for water and slurry flowing through 200 mm × 50 mm cross-section rectangular duct and 105 mm diameter pipe at different efflux concentrations.



(a) $C_{vf} = 4$ %

(b) $C_{vf} = 8.1$ %

(c) $C_{vf} = 12.8$ %

(d) $C_{vf} = 19.1$ %

(e) $C_{vf} = 26$ %

Fig. 10. Variation of additional friction factor due to solids f_{hs} with Reynolds number Re_w of slurry flow through 200 mm × 50 mm cross-section rectangular duct and 105 mm diameter pipe at different efflux concentrations.

where f_h, f_{hw} and f_{hs} are slurry friction factor, friction factor due to water and additional friction factor due to presence of solids. f_{hs} at different Reynolds number could be obtained by subtracting f_{hw} from f_h. Fig. 10 shows the variation of f_{hs} with Reynolds number for pipe and rectangular duct at different efflux concentrations. It is observed that f_{hs} for rectangular duct is always less than that for pipe. Analysis of experimental results shown in Figs. 4, 5, 9 and 10 shows that f_h for rectangular duct becomes less than that for pipe for slurry flow, because, in spite of a slight increase in f_{hw}, there is a substantial decrease in f_{hs}. Fig. 10 shows that the substantial decrease in f_{hs} for a slight increase in f_{hw} is more prominent at the lower efflux concentrations (namely 4% and 8.1%) in the higher Reynolds number range due to increased effect of secondary flow on solids concentration profiles in rectangular duct. However, as the efflux concentration increases, f_{hs} does not increase that substantially with slight increase in f_{hw}. The effect of particle diffusivity on solids concentration profiles which increases exponentially with the efflux concentration (Fig. 8) is more prominent at higher efflux concentrations in both the rectangular duct and pipe.

6. Conclusions

Following conclusions have been drawn on the basis of the present study:

1. Solids concentration profiles were found to be a function of particle size, velocity of flow and efflux concentration of slurry. Solids concentration varied with the vertical position, except for particle size of 38 μm.
2. Pressure drop at different Reynolds number, due to the flow of slurry through rectangular duct having 200 mm × 50 mm cross-section are always less than that for the circular pipe having diameter of 105 mm.
3. Decrease in pressure drop, in case of rectangular duct, reduces with increase in efflux concentration.

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